Engineering Stress and Strain (Review)

**Stress**: Force per unit area (N/m\(^2\) is a Pascal)

\[ \sigma = \frac{F}{A_0} \]

**Strain**: The ratio of the change in length over the initial length of a specimen (unit is m/m)

\[ \varepsilon = \frac{\Delta l}{l_0} \]

- **Tension**
- **Compression**
- **Shear**
- **Torsion**
True Stress & Strain

Points of Necking

True Stress: Load over instantaneous cross-sectional area

\[ \sigma_{True} = \frac{F}{A_i} \]

\[ \epsilon_{True} = \ln\left(\frac{l_i}{l_0}\right) \]

Relating True and Engineering Stress and Strain (before necking & no volume change):

\[ \sigma_{True} = \sigma (1 + \epsilon) \]

\[ \epsilon_{True} = \ln (1 + \epsilon) \]
Tensile Behavior

\[ \sigma = E \varepsilon \]

E – Young’s Modulus of Elasticity
(For Tension/Compression)

\[ \tau = G \gamma \]

G – Shear Modulus
\( \tau \) – Shear Strain (For Shear)
\( \gamma \) - Shear Strain

Poisson's Ratio

\[ \nu = - \frac{\varepsilon_x}{\varepsilon_z} = - \frac{\varepsilon_y}{\varepsilon_z} \]
Elastic Vs. Plastic Deformation

Elastic Deformation: Deformation which is completely reversible; stress proportional to strain
Interatomic bonds stretch

Plastic Deformation: Permanent deformation; stress not proportional to strain
Interatomic bonds start to break

Yield Strength – by convention known as the point, shown above.
Deformation - Dislocations

Interatomic bonds break and reform – atoms move in direction of stress

Dislocation Interaction

Opposites attracts and Identical objects repel.

Slip Systems

Usually move along a close-packed direction

Strengthening Materials

- Grain Size Reduction
- Solid-solution
- Strain (Work) Hardening
Annealing

- Recovery
- Recrystallization
- Grain growth

Creep

- Tensile strength
- Ductility

- Recovery
- Recrystallization
- Grain growth

- Cold-worked and recovered grains
- New grains

- Creep strain, $\varepsilon$
  - Primary
  - Secondary
  - Tertiary
  - Rupture

- Time, $t$
  - $t_f$

Recovery, grain growth, recrystallization.
Fracture Mechanism

This diagram show a crack which is being opened.

As a crack is being opened, the breaking of bond induces two things:
- reduces the total strain energy
- increases the total surface energy

As the crack begins to start, the surface energy is greater than the strain energy and the surface stays in tact.

As more strain is felt, the strain energy becomes larger than the surface energy and the crack beings to grow.
Fracture

Brittle
- little plastic deformation
- low energy absorption

Ductile
- significant plastic deformation
- high energy absorption

Tensile Strength
Fracture Strength
Fracture Mechanics

\[ K_c = Y \sigma_c \sqrt{\pi a} \]

\[ \sigma_c = \left( \frac{2 E \gamma_s}{\pi a} \right)^{1/2} \]

\[ K_t = \frac{\sigma_m}{\sigma_o} = 2 \left( \frac{a}{\rho_t} \right)^{1/2} \]

\( a \) - ½ crack length

**\( K_c \) – fracture toughness**
(materials resistance to brittle fracture)

**\( Y \) – dimensionless parameter**
(depends on crack geometry)

**\( \sigma_c \) – critical stress**

E – modulus of elasticity

**\( \gamma_s \) – specific surface energy**

**\( K_t \) – stress concentration factor**

**\( \rho_t \) – radius of curvature of crack tip**
Hardness, Toughness and Strength
What is the Difference?

Hardness: A material's resistance to plastic deformation

eg. Diamond is a very HARD substance

Tensile Strength: Amount of Stress a material may withstand

eg. Stainless Steel is very STRONG

Toughness: A material's ability to absorb energy up to fracture

eg. Titanium alloys are very TOUGH materials


http://www.cee.uiuc.edu/paulino/Pages/Publications/Papers/RepresentativeResults/efm1r.htm
Fatigue

**Fatigue**: Failure at a stress level considerably lower than tensile strength.

estimated to comprise approximately 90% of all metallic failures and is catastrophic and insidious.

***Fatigue limit***: below which fatigue failure will not occur.
Fracture Mechanics Example

For a good example of stress concentration, let's look at three different sections. The first section is flawless and has a force applied in the middle. The next two have flaws with forces acting at these flaws. The first has the force acts right at the flaw while the other section has a force acting on the opposite side of the flaw. Which section will break first?

![Diagram of three sections with forces at flaws](image)

Note: Each bar has the same dimensions and the flaws on the last two have the same dimensions.

Discuss with yourselves and try to picture where the stress concentrations are and how they are acting on the sections. If you know the answer, explain it in terms of the stress concentrations. Have a diagram on the board if it helps. Nails with flaws with them will be available to test out your theory.
Experiment – Deformation

Many different situations give way to different relationships of properties. To show this, let's take a look at a foam material (cork) and a natural polymer (balsa wood). Let us compare and contrast these 2 materials:

Instructions:
1. Obtain the following materials:
   - vice
   - balsa wood
   - cork
   - calculator

2. Now take your 2 materials and compress them using a vice along their longest direction. One breaks catastrophically and the other doesn't. Which one, why? (Note: If you have difficulty with the vice, you can use your fingers!)

3. In the Balsa wood, you can see defined grains. Using the vice, compress the Balsa wood again but this time with the stress parallel to the grain of the Balsa. How does Balsa wood differ when tested parallel to the grain and perpendicular to the grain.

4. Discuss with your group what each material's engineering stress-strain curve would look like.
Experiment – Mechanical Properties

Instructions:
1. Break off into groups of 5 and get the following:
   - 1 meter ruler
   - tape
   - scissors
   - nails (~33.5g/each)
   - elastic band
   - bucket w/string (~37.8g)
   - stand with heavy base

2. Measure the initial length and area of the rubber band.

3. Attach the rubber band to the stand with tape and put the bucket at the bottom of the band. Attach it with tape as well. To make this experiment faster, use shorter and thinner bands.

4. Stretch the rubber band until it breaks with adding weights. Take length measurements of the band after each weight put in. Plot a Stress vs. Strain curve.

5. Using these measurements, calculate the breaking strength of the rubber band. Looking at your data, is this elastic band completely elastic?


To maximize your time in this experiment split up the following jobs:
   - placing weights
   - measuring the elastic
   - taking down values
   - convert values to stress and strain
   - plotting the stress/strain curve
Experiment - Mechanical Properties
Quiz

In the early to mid 20th century, it was not uncommon for large ships to break in half. There could be many reasons why this is so such as material composition or ship structure. Let's say you were the engineer doing the failure analysis on one of the broken ships. You find that there was a 1m diameter window where the crack was found. This window was placed near the top of the ship. After doing material analysis, you find that the hull of the ship was made of Grey Cast Iron (Modulus of elasticity of 165 GPa) and you estimate that the surface tension was 10 J/m².

a) Calculate the stress (in MPa) at which the crack began to propagate.

b) What could be done to this boat so that it would not fail in such a manner? Note that cast iron performs MUCH better in compression than in tension. Think back to a qualitative example you just looked at in the tutorial also the forces a boat encounters at sea (large waves).